Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Advanced Real Analysis

Subject Code: 5SC03ARA1 **Branch: M.Sc. (Mathematics)**

Semester: 3 **Date: 21/11/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. True/False: Every $\sigma-$finite measure is a finite. **(01)**

b. Let $(X,\mathcal{A})$ be a measurable space if $\lambda \,\&\, \mu$ are two measure on $(X,\mathcal{A})$ with $\lambda \perp \mu$ and $\lambda << \mu$ then $\lambda = 0$. **(02)**

c. Define: Absolutely continuous measure **(02)**

d. Write chain rule for Radon-Nikodym derivative. **(02)**

Q-2 **Attempt all questions** **(14)**

a. State and prove Monotone convergence theorem. **(10)**

b. Let $(X,\mathcal{A},\mu)$ be a measure space and $E_i \in \mathcal{A},\ E_i \supset E_{i+1}$. Let $\mu(E_1) < \infty$ then $\mu\left(\bigcap_n E_n\right) = \lim_{n\to\infty} \mu(E_n)$. **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a. State and prove Lusin's theorem. **(08)**

b. Let S be a non-negative simple measurable function on a measure space $(X,\mathcal{A},\mu)$. If $\rho(E) = \int_E S\, d\mu,\ E \in \mathcal{A}$ then ρ is a measure on $(X,\mathcal{A})$. **(04)**

c. State Fatou's lemma. **(02)**

Q-3 **Attempt all questions** **(14)**

a. State and prove Hahn-Decomposition theorem. **(07)**

	b.	State and prove Lebesgue Dominated Convergence theorem.	**(05)**
	c.	Define: Mutually singular measures	**(02)**

OR

Q-3		**Attempt all questions**	**(14)**
	a.	State and prove Jordan Decomposition theorem.	**(08)**
	b.	Prove that countable union of positive set is positive set.	**(04)**
	c.	Define: Signed measure	**(02)**

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	Give an example of self-conjugate number.	**(01)**
	b.	State Tonelli's theorem.	**(02)**
	c.	Define: Locally compact	**(02)**
	d.	Define: Norm in $L^p(\mu)$	**(02)**
Q-5		**Attempt all questions**	**(14)**
	a.	State and prove Radon-Nikodym theorem.	**(10)**
	b.	If $f, g \in L^p(\mu)$ then prove that $\|f+g\|_p \le \|f\|_p + \|g\|_p$.	**(04)**

OR

Q-5		**Attempt all questions**	**(14)**
	a.	State and prove Lebesgue Decomposition theorem.	**(08)**
	b.	State and prove Holder's inequality.	**(06)**
Q-6		**Attempt all questions**	**(14)**
	a.	State and prove Caratheodory theorem.	**(07)**
	b.	Let $(X, \mathcal{B}, \mu)$ be a finite measure space and g be an integrable function such that for some constant M, $\left|\int g\phi \, d\mu\right| \le M \|\phi\|_p$ for all simple functions ϕ then $g \in L^q$.	**(05)**
	c.	Define: Product measure space	**(02)**

OR

Q-6		**Attempt all Questions**	**(14)**
	a.	State and prove Riesz representation theorem.	**(09)**
	b.	Let X be a normed space then X is complete iff every absolutely summable series is summable.	**(05)**